Eur Phys J C **32**, s02, s191–s204 (2004)
Digital Object Identifier (DOI) 10.1140/epjcd/s2003-01-017-1

EPJ C direct
electronic only

Global analysis of $D \to PV$ decays and SU(3) flavor symmetry breaking effects

M. Zhong[1], Y.L. Wu[1], and W.Y. Wang[2]

[1] Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China
[2] Department of Physics, Tsinghua University, Beijing 100084, China

Received: 20 Jul 2003 / Accepted: 8 Dec 2003 /
Published Online: 17 Dec 2003 –

Abstract. We investigate in detail both the Cabibbo-allowed and singly Cabibbo-suppressed $D \to PV$ decays based on the diagrammatic decomposition in the factorization formalism. Two sets of solutions discarded in the literature are picked up and discussed carefully. It is found that one of these solutions can provide satisfactory explanation in a natural manner on the process $D^+ \to \overline{K}^0 K^{*+}$ which is thought to be a puzzle. The relations $E_V + E_P = 0$ and $A_V + A_P = 0$ are badly broken, which indicates that the exchange and annihilation diagrams may receive contributions from more sources other than the $q\bar{q}$ intermediate state interactions. It is shown that, to have a consistent explanation to the experimental data with reasonable values for the parameters a_1 and a_2, the SU(3) symmetry breaking effects have to be considered. The SU(3) flavor symmetry breaking effects due to mass factors and due to formfactors and decay constants are analyzed in detail.

1 Introduction

A great number of precise experimental data on charmed meson nonleptonic two body decays have been accumulated to guide, constrain and test the theoretical studies. The theoretical settlement of this transition type generally appeals to factorization hypothesis. Empirically, nonfactorizable correction should be considered. But the nonfactorizable effects are relatively hard to be calculated in charmed meson decays comparing with bottom meson decays. Phenomenological models based on all kinds of symmetries are of importance to explore the decay zoos. But in some cases, the breaking effects due to masses and due to formfactors and decay constants can significantly be enhanced.

In the quark diagrammatic scenario, all two-body nonleptonic weak decays of charmed mesons can be expressed in terms of six distinct quark-graph contributions [1,2]: (1) a color-favored tree amplitude T, (2) a color-suppressed tree amplitude C, (3) a W-exchange amplitude E, (4) a W-annihilation amplitude A, (5) a horizontal W-loop amplitude P and (6) a vertical W-loop amplitude D. The P and D diagrams play little role in practice because the CKM matrix elements have the relation $V_{cs}V_{us} \approx -V_{cd}V_{ud}$ which will result in these diagrams cancelling each other. Based on SU(3) flavor symmetry, the T, C, E and A amplitudes were fitted from the measured D meson decay modes [3,4]. These amplitudes help one to understand the generality of charmed meson decays. But since SU(3) flavor symmetry breaking effects appear to be important as pointed out in [5], these fitted data can not describe the specific properties in certain decay modes. Most importantly, two sets of solutions considered as disfavored in [3,4] will be shown to be reasonable in the formalism of factorization. It is of interest to investigate how the SU(3) flavor symmetry breaking will influence on charmed meson transitions and what these sets of solutions can provide us.

In this paper, we will pay attention to the SU(3) flavor symmetry breaking effects in charmed meson decays to a pseudoscalar and a vector meson by using the quark-graph description. Since factorization formalism reflects SU(3) flavor symmetry breaking effects, it is convenient to take such a formalism to investigate the symmetry breaking levels. Firstly by dividing these diagrams into factors including SU(3) flavor symmetry breaking effects and introducing parameters describing the overall properties, we arrive at two sets of solutions for the parameters from fitting experimental data without adding any assumption in advance. Using the fitting parameters as inputs, we are led to predictions for the branching ratios of other decay modes which are expected to be measured in the future. Of interest, one solution for a set of parameters that is regarded to be the disfavored solutions in [3,4] can provide satisfactory explanation to the process $D^+ \to \overline{K}^0 K^{*+}$. So that the puzzle in the process [6] disappears and it is not necessary to introduce new physics. It is also shown that some relations containing contribution from $A_P - A_V$ are no longer reliable due to large SU(3) flavor symmetry breaking effects. Except those relations, the breaking effects due to masses and due to formfactors and decay constants can be as large as 12% and 16% respectively in one set of solutions. For some processes the total breaking amount can reach to 21%, when the two symmetry breaking effects due to masses and due to formfactors and decay constants become to be coherently added. The breaking effects due to masses and due to formfactors and decay constants can add up to 18% and 24% respectively in another set of solutions, which can lead to the total breaking amount up to 43% in some processes. In addition, it is worth stressing that $D \to PV$ decay modes receive large nonfactorizable contributions. The relations $E_P = -E_V$ and $A_P = -A_V$, resulted from the assumption that the contributions of all these four diagrams come from $q\overline{q}$ intermediate state interactions and used as inputs in [4], are broken down. In contrast, E_P and A_P seem to be close to E_V and A_V respectively, which implies that more sources of contributions other than the $q\overline{q}$ intermediate state interactions might be taken into account in these diagrams.

In methodology, there are some differences between the [4] and the present paper. SU(3) invariant amplitudes were extracted out from the Cabibbo-allowed two-body D meson decays alone in [3,4]. However, only from the Cabibbo-allowed two-body decays, there are not enough decay modes as inputs and constraints to extract all the amplitudes. Some assumptions about E and A amplitudes have to be made and some knowledges on the color-favored tree diagrams T and the color-suppressed tree diagrams C have to be taken from the factorization hypothesis. In this way, two sorts of inconsistency may arise. Firstly, when the results presented in [4] are expanded to the Cabibbo-suppressed modes, some of the predicted branching ratios are inconsistent with the experimental data. Secondly, since the factorization hypothesis reflects SU(3) flavor symmetry breaking effects, there may bring about some inconsistency in logic when using its conclusions in the SU(3) symmetry method. The second inconsistency might be enhanced when the SU(3) symmetry breaking effects are large enough. One will see that this sort of inconsistency does appear in the $D \to PV$ decays, which implies that the SU(3) symmetry breaking effects due to mass factors and due to formfactors and decay constants are too large to be ignored. In our method, by combining the Cabibbo-allowed two-body decays with the Cabibbo-suppressed ones, we have enough experimental inputs to extract all of the parameters without any assumptions. Moreover, there are additional experimental data to constrain our solutions. In this way, we obtain results which are significantly different from that given in [3,4].

The paper is organized as follows. In Sect. 2, we list the flavor decomposition of the corresponding mesons and present the quark-diagram description for the decay modes which have been observed. In Sect. 3, the parameterized formalism based on factorization is introduced to investigate the processes. We then make a detailed numerical analysis in Sect. 4 for the parameters and present predictions for twenty one decay modes. The way to extract the parameterized a_is from the invariant amplitudes is studied in Sect. 5. The SU(3) flavor symmetry breaking effects are discussed in Sect. 6. A short summary is given in the last section.

2 Notation and quark-diagram formalism

We adopt the following quark contents and phase conventions which have been widely used [2,3,4,7].

- *Charmed mesons*: $D^0 = -c\overline{u}$, $D^+ = c\overline{d}$, $D_s^+ = c\overline{s}$;
- *Pseudoscalar mesons P*: $\pi^+ = u\overline{d}$, $\pi^0 = (d\overline{d} - u\overline{u})/\sqrt{2}$, $\pi^- = -d\overline{u}$, $K^+ = u\overline{s}$, $K^0 = d\overline{s}$, $\overline{K}^0 = s\overline{d}$, $K^- = -s\overline{u}$, $\eta = (-u\overline{u} - d\overline{d} + s\overline{s})/\sqrt{3}$, $\eta' = (u\overline{u} + d\overline{d} + 2s\overline{s})/\sqrt{6}$;
- *Vector mesons V*: $\rho^+ = u\overline{d}$, $\rho^0 = (d\overline{d} - u\overline{u})/\sqrt{2}$, $\rho^- = -d\overline{u}$, $\omega = (u\overline{u} + d\overline{d})/\sqrt{2}$, $K^{*+} = u\overline{s}$, $K^{*0} = d\overline{s}$, $\overline{K}^{*0} = s\overline{d}$, $K^{*-} = -s\overline{u}$, $\phi = s\overline{s}$.

In the above notations, u, d and s quarks transform as a triplet of flavor SU(3) group, and $-\overline{u}$, $\overline{d}$ and $\overline{s}$ as an antitriplet, so that mesons form isospin multiplets without extra signs. In general, the $\eta\eta'$ mixing are defined as

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} \tag{1}$$

with $\eta_0 = (u\overline{u}+d\overline{d}+s\overline{s})/\sqrt{3}$ and $\eta_8 = (-u\overline{u}-d\overline{d}+2s\overline{s})/\sqrt{6}$. For convenience, we have taken the mixing parameter as $\phi = 19.5^\circ = \sin^{-1}(1/3)$ which is close to the value $\phi = 15.4^\circ$ extracted from experiment [8].

The partial width Γ for $D \to PV$ decays is expressed in terms of an invariant amplitude $\mathcal{A}$ as

$$\Gamma(D \to PV) = \frac{p^3}{8\pi M_D^2}|\mathcal{A}|^2 \tag{2}$$

where

$$p = \frac{\sqrt{(M_D^2 - (m_P + m_V)^2)(M_D^2 - (m_P - m_V)^2)}}{2M_D}$$

denotes the center-of-mass 3-momentum of each final particle.

We summarize in Table 1 the quark-diagram representation, the branching ratios[9] and invariant amplitudes of the transition modes which are used as inputs to fix the parameters involved in our considerations. In the numerical analysis, we will use the masses and lifetimes for the charmed mesons $M_{D^+} = (1869.3 \pm 0.5)$MeV with $\tau(D^+) = (1051 \pm 13)$fs, $M_{D^0} = (1864.5 \pm 0.5)$MeV with $\tau(D^0) = (411.7 \pm 2.7)$fs, and $M_{D_s^+} = (1968.5 \pm 0.6)$MeV with $\tau(D_s^+) = (490 \pm 9)$fs [9]. In quark-diagram representation the subscript P or V are assigned to T and C, which are induced by $c \to q_3 q_1 \overline{q}_2$ with the spectator quark containing in pseudoscalar or vector final meson. The subscript P or V are labelled to E and A graphs which are dominated by the weak process $c\overline{q}_1 \to q_2\overline{q}_3$ when the final antiquark $\overline{q}_3$ stays in the pseudoscalar or vector meson. S is added before E or A to distinguish the exchange or annihilation graph involving in final singlet state contributions which result from disconnected graphs. The total contributions of the SE and SA graphs involving in π^0 and ρ^0 mesons are equal to zero because their contributions resulting from $u\overline{u}$ and $-d\overline{d}$ offset each other due to the isospin SU(2) symmetry. In the numerical analysis, we

Table 1. Quark-diagram presentation, branching ratios and invariant amplitudes of charmed mesons decaying to one pseudoscalar and one vector meson. Primes are added to the representation to denote the singly Cabibbo-suppressed processes

Initial Meson	Decay Mode	Representation	$\mathcal{B}$ [9] (%)	p^* (MeV)	$\lvert\mathcal{A}\rvert$ (10^{-6})
D^0	$K^{*-}\pi^+$	$T_V + E_P$	6.0 ± 0.5	711	4.83 ± 0.20
	$K^-\rho^+$	$T_P + E_V$	10.2 ± 0.8	678	6.76 ± 0.26
	$\overline{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}(C_P - E_P)$	2.8 ± 0.4	709	3.31 ± 0.24
	$\overline{K}^0\rho^0$	$\frac{1}{\sqrt{2}}(C_V - E_V)$	1.47 ± 0.29	676	2.57 ± 0.25
	$\overline{K}^{*0}\eta$	$\frac{1}{\sqrt{3}}(C_P + E_P - E_V + SE_V)$	1.8 ± 0.4	580	3.59 ± 0.40
	$\overline{K}^0\omega$	$-\frac{1}{\sqrt{2}}(C_V + E_V)$	2.2 ± 0.4	670	3.20 ± 0.29
	$\overline{K}^0\phi$	$-E_P - SE_P$	0.94 ± 0.11	520	3.05 ± 0.18
D^+	$\overline{K}^{*0}\pi^+$	$T_V + C_P$	1.92 ± 0.19	712	1.71 ± 0.08
	$\pi^+\phi$	$C'_P - SA'_P$	0.61 ± 0.06	647	1.113 ± 0.055
	$\overline{K}^0\rho^+$	$T_P + C_V$	6.6 ± 2.5	680	3.40 ± 0.64
	$\pi^+\rho^0$	$-\frac{1}{\sqrt{2}}(T'_V + C'_P - A'_P + A'_V)$	0.104 ± 0.018	769	0.355 ± 0.031
	$K^+\overline{K}^{*0}$	$T'_V - A'_V$	0.42 ± 0.05	610	1.008 ± 0.060
D_s^+	$\overline{K}^{*0}K^+$	$C_P + A_V$	3.3 ± 0.9	682	3.69 ± 0.50
	$\overline{K}^0K^{*+}$	$C_V + A_P$	4.3 ± 1.4	683	4.20 ± 0.68
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}(A_V - A_P)$	$0.06^{\ddagger}(< 0.07)$	822	$0.37^{\ddagger}(< 0.40)$
	$\pi^+\phi$	$T_V + SA_P$	3.6 ± 0.9	712	3.61 ± 0.45

‡ The central value of the E791 experiment [30].

will assume that the contributions of the SE_P and SE_V graphs involving in ω and ϕ mesons are negligibly small since they seem not to contradict with the Okubo-Zweig-Iizuka rule. But the amplitude SA_V seems to play an important role in the $D_s^+ \to \rho^+\eta$ and $D_s^+ \to \rho^+\eta'$ processes [10]. In the ideal mixing case, the process $D_s^+ \to \pi^+\omega$ has the amplitude representation as $\frac{1}{\sqrt{2}}(A_V + A_P + 2SA_P)$. Since ω has the similar quark structure as comparison with η and η', we assume that SA_P also has important contribution in $D_s^+ \to \pi^+\omega$. In the present paper, we shall not consider the processes which receive contributions from SA_V and SA_P diagrams resulting from the final state particles η, η' or ω . The sign flips in the presentation of some relevant Cabibbo-favored modes come from the quark contents of final light mesons. In the singly Cabibbo-suppressed modes, the sign flips may come either from the quark contents of the final light mesons or from the CKM matrix element $V_{cd}V_{ud}$ since $V_{cs}V_{us} \approx -V_{cd}V_{ud}$ and we choose $V_{cs}V_{us}$ in the calculations. In Table 1 and Table 4, a prime is added to the diagrams of singly Cabibbo-suppressed modes to distinguish them from the Cabibbo-favored ones.

3 Flavor SU(3) symmetry breaking description in factorization formalism

To investigate the SU(3) flavor symmetry breaking terms, we take the formalism of factorization approach. In [11], a detailed discussion on the factorization has been applied to nonleptonic two-body bottom meson decays. For $D \to PV$ decays, amplitudes can be written in the factorized form as [12,13]

$$T_V = \frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_{T_V} 2 f_P m_{D_i} A_0^{D_i \to V}(m_P^2), \qquad (3)$$

$$T_P = \frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_{T_P} 2 f_V m_{D_i} F_1^{D_i \to P}(m_V^2), \qquad (4)$$

$$C_V = \frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_{C_V} 2 f_P m_{D_i} A_0^{D_i \to V}(m_P^2), \qquad (5)$$

$$C_P = \frac{G_F}{\sqrt{2}} V_{q_1q_2} V^*_{cq_3} a_{C_P} 2 f_V m_{D_i} F_1^{D_i \to P}(m_V^2), \qquad (6)$$

$$E_V = \frac{G_F}{\sqrt{2}} V_{q_1q_3} V^*_{cq_2} a_{E_V} 2 f_{D_i} m_{D_i} A_0^{PV}(m_{D_i}^2), \qquad (7)$$

$$E_P = \frac{G_F}{\sqrt{2}} V_{q_1q_3} V^*_{cq_2} a_{E_P} 2 f_{D_i} m_{D_i} A_0^{PV}(m_{D_i}^2), \qquad (8)$$

$$A_V = \frac{G_F}{\sqrt{2}} V_{q_2q_3} V^*_{cq_1} a_{A_V} 2 f_{D_i} m_{D_i} A_0^{PV}(m_{D_i}^2), \qquad (9)$$

$$A_P = \frac{G_F}{\sqrt{2}} V_{q_2q_3} V^*_{cq_1} a_{A_P} 2 f_{D_i} m_{D_i} A_0^{PV}(m_{D_i}^2), \qquad (10)$$

where D_i denotes $D^{\pm}$, D_0 or D_s. V_{ud}, V_{us} and V^*_{cs} are the relevant CKM matrix elements. F_1 and A_0 are formfactors defined in the following formalism

$$\langle P(p)|\bar{q}\gamma^\mu c|D(p_D)\rangle = \left[(p_D + p)_\mu - \frac{m_D^2 - m_P^2}{q^2} q^\mu\right] F_1(q^2) + \frac{m_D^2 - m_P^2}{q^2} q^\mu F_0(q^2), \qquad (11)$$

$$
\begin{aligned}
\langle V(p)|\bar{q}\gamma^\mu(1-\gamma^5)c|D(p_D)\rangle &= -i(m_D+m_V)A_1(q^2) \\
&(\epsilon^{*\mu} - \frac{\epsilon^*\cdot q}{q^2}q^\mu) + i\frac{A_2(q^2)}{m_D+m_V} \\
&(\epsilon^*\cdot q)((p_D+p)^\mu - \frac{m_D^2-m_V^2}{q^2}q^\mu) \\
&-i\frac{2m_V}{q^2}(\epsilon^*\cdot q)A_0(q^2)q^\mu \\
&-\frac{2V(q^2)}{m_D+m_V}\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p_{D\beta}p_\gamma, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
\langle P(p_P)V(p_V)|j^\mu|0\rangle &= -i(m_P+m_V)A_1^{PV}(q^2) \\
&(\epsilon^{*\mu} - \frac{\epsilon^*\cdot q}{q^2}q^\mu) + i\frac{A_2^{PV}(q^2)}{m_P+m_V} \\
&(\epsilon^*\cdot q)((p_P-p_V)^\mu - \frac{m_P^2-m_V^2}{q^2}q^\mu) \\
&-i\frac{2m_V}{q^2}(\epsilon^*\cdot q)A_0^{PV}(q^2)q^\mu \\
&-\frac{2V^{PV}(q^2)}{m_P+m_V}\epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha p_{P\beta}p_{V\gamma}, \qquad (13)
\end{aligned}
$$

with $q = p_D - p$. f_P and f_V are decay constants defined as

$$\langle P(p)|\bar{q}_1\gamma^\mu\gamma_5 q_2|0\rangle = -if_P p^\mu, \qquad (14)$$
$$\langle V(p)|\bar{q}_1\gamma^\mu q_2|0\rangle = f_V m_V \epsilon^\mu. \qquad (15)$$

It is obvious that SU(3) flavor symmetry breaking effects are embodied in equations (3)-(10). These effects result from masses (m_P,m_V,m_{D_i}), decay constants and form factors.

In naive factorization hypothesis, one has the following equalities

$$a_{T_V} = a_{T_P} = a_{A_V} = a_{A_P} = a_1(\mu), \qquad (16)$$
$$a_{C_V} = a_{C_P} = a_{E_V} = a_{E_P} = a_2(\mu), \qquad (17)$$

with

$$a_1(\mu) = c_1(\mu) + \frac{1}{N_c}c_2(\mu), \qquad (18)$$
$$a_2(\mu) = c_2(\mu) + \frac{1}{N_c}c_1(\mu), \qquad (19)$$

denoting the relations between quantities $a_{1,2}$ and Wilson coefficients $c_{1,2}$. N_c is the number of colors. In charmed decays, large-N_c is justified because it can greatly improve the discrepancy between theory and experiment. μ is the renormalization scale at which c_1 and c_2 are evaluated. So a_1 and a_2 are common real quantities of a certain process in quark level. To be more explicit, for decay modes induced by $c \to s$ transition, a_1 and a_2 are invariant among all modes in naive factorization hypothesis.

However, naive factorization approach fails to describe charmed meson decays, particularly for the decay modes which involve in the color-suppressed diagrams due to the smallness of $|a_2|$. Furthermore, the coefficients a_1 and a_2 in Eqs.(16) and (17) depend on the renormalization scale and γ_5 scheme. On the other hand, it is also necessary to consider the nonfactorizable corrections which involve in hard spectator interactions, final state interactions and resonance effects etc. In this case, one can express a_1 and a_2 in the form

$$a_1(\mu) = c_1(\mu) + (\frac{1}{N_c} + \chi_1(\mu))c_2(\mu), \qquad (20)$$
$$a_2(\mu) = c_2(\mu) + (\frac{1}{N_c} + \chi_2(\mu))c_1(\mu), \qquad (21)$$

with $\chi_1(\mu)$ and $\chi_2(\mu)$ terms denoting the nonfactorizable effects. Furthermore, with nonfactorization corrections the equalities (16) and (17) are not yet satisfied because each a_i should contain term from corrections. The nonfactorization corrections can also bring phase differences among these coefficients, and then a_is ($i = T_{V,P}, C_{V,P}, E_{V,P}$ and $A_{V,P}$) turn into complexes. In general, explicit calculations of total nonfactorizable corrections are not yet possible. We shall take all a_is as independent complex parameters and assume that the corrections do not depend on individual decay process at certain scale. In other words, a_is do not include SU(3) flavor symmetry violation contributions. It is supposed that mass factors, decay constants and formfactors have taken on the whole SU(3) symmetry breaking effects.

4 Numerical analysis and results

The explicit evaluation of the relevant formfactors in the factorization formula (3)-(10) are not yet available because of the nonperturbative long distance effects of QCD. But some reasonable methods, such as QCD sum rules[14,15], lattice simulations [16,17] and phenomenological quark model [18,19], have been developed to estimate the long distance effects to rather high certainties. The formfactors of D mesons decaying to light mesons have been widely discussed in [20,21,22,23,24,25]. In our research, we shall use the results obtained by Bauer, Stech and Wirbel [20] based on the quark model. They have been found to be rather successful in describing a number of processes concerning D mesons. The values of the relevant formfactors evaluated at $q^2 = 0$ are listed in Table 2. For the dependence on q^2, the formfactors are assumed to behave as a monopole dominance

$$D \to P: \quad F_1(q^2) = \frac{F_1(0)}{1 - q^2/m_{F^*}^2}, \qquad (22)$$
$$D \to V: \quad A_0(q^2) = \frac{A_0(q^2)}{1 - q^2/m_F^2}, \qquad (23)$$

where m_F and m_{F^*} are the pole masses given in Table 2.

The formfactors A_0^{PV} involving in the exchange and annihilation amplitudes are hard to relate directly to experimental measurements. They are greatly suppressed at large momentum transfer $q^2 = m_{D_i}^2$, which leads to the smallness of the contributions from the factorizable exchange and annihilation diagrams. The main contributions of these diagrams may result from the unfactorizable forms. Through intermediate states, these diagrams relate

Table 2. Relevant formfactors at zero momentum transfer for $D \to P$ and $D \to V$ transitions and pole masses in BSW model

Decay	$D \to \pi$	$D \to \rho(\omega)$	$D \to K$	$D \to K^*$	$D_s \to K$	$D_s \to K^*$	$D_s \to \phi$	$D \to \eta/\eta'$	$D_s \to \eta/\eta'$
F_1	0.692		0.762		0.643			0.681/0.655	0.723/0.704
A_0		0.669		0.733		0.634	0.700		
$m_F(GeV)$		1.87		1.97		1.87	1.97		
$m_{F^*}(GeV)$	2.01		2.11		2.01			2.01	2.11

Table 3. Values of decay constants in MeV

f_π	f_K	f_8	f_0	f_D	f_{D_s}	f_ρ	f_{K^*}	f_ω	f_ϕ	f_{D^*}	$f_{D_s^*}$
134	158	168	157	200	234	210	214	195	233	230	275

Table 4. Predicted branching ratios for charmed mesons decaying to one pseudoscalar and one vector meson

Meson	Decay Mode	Representation	Experimental $\mathcal{B}(\times 10^{-2})$	Present $\mathcal{B}(\times 10^{-2})$ case (I)	Present $\mathcal{B}(\times 10^{-2})$ case (II)	$\mathcal{B}(\times 10^{-2})$ in [31]
D^0	K^+K^{*-}	$T'_V + E'_P$	0.20 ± 0.11	0.25	0.25	0.30
	K^-K^{*+}	$T'_P + E'_V$	0.38 ± 0.08	0.42	0.41	0.43
	$K^0\overline{K}^{*0}$	$E'_V - E'_P$	< 0.17	0.03	0.02	0.062
	$\overline{K}^0K^{*0}$	$E'_P - E'_V$	< 0.09	0.03	0.02	0.064
	$\pi^0\phi$	$\frac{1}{\sqrt{2}}(C'_P + SE'_P)$	< 0.14	0.12	0.12	0.11
	$\overline{K}^{*0}\eta'$	$-\frac{1}{\sqrt{6}}(C_P + E_P + 2E_V + 4SE_V)$	< 0.10	0.008	0.01	0.004
	$\eta\phi$	$\frac{1}{\sqrt{3}}(C'_P - 2SE'_P + SE'_V)$	< 2.8	0.04	0.04	0.090
	$\pi^+\rho^-$	$-(T'_V + E'_P)$	—	0.34	0.34	0.57
	$\pi^-\rho^+$	$-(T'_P + E'_V)$	—	0.64	0.65	0.69
	$\pi^0\rho^0$	$\frac{1}{2}(C'_P + C'_V - E_P - E_V)$	—	0.12	0.24	0.12
	$\pi^0\omega$	$\frac{1}{2}(C'_V - C'_P + E'_P + E'_V + 2SE'_P)$	—	0.24	0.20	0.014
	$\eta\omega$	$-\frac{1}{\sqrt{6}}(C'_P + 2C'_V + SE'_V + 4SE'_P)$	—	0.0007	0.07	0.20
	$\eta'\omega$	$\frac{1}{2\sqrt{3}}(C'_P - C'_V + 4SE'_V - 2SE'_P)$	—	0.003	0.0006	0.0001
	$\eta\rho^0$	$\frac{1}{\sqrt{6}}(2C'_V - C'_P - SE'_V)$	—	0.11	0.005	0.020
	$\eta'\rho^0$	$\frac{1}{2\sqrt{3}}(C'_V + C'_P + 4SE'_V)$	—	0.0005	0.003	0.008
D^+	$\overline{K}^0K^{*+}$	$T'_P - A'_P$	3.1 ± 1.4	3.47	0.59	1.71
	$\pi^0\rho^+$	$-\frac{1}{\sqrt{2}}(T'_P + C'_V + A'_P - A'_V)$	—	0.23	0.36	0.44
D_s^+	π^+K^{*0}	$-(T'_V - A'_V)$	0.65 ± 0.28	0.51	0.21	0.29
	$K^+\rho^0$	$-\frac{1}{\sqrt{2}}(C'_P + A'_P)$	< 0.29	0.16	0.08	0.29
	$K^0\rho^+$	$-(T'_P - A'_P)$	—	2.73	0.27	1.39
	π^0K^{*+}	$-\frac{1}{\sqrt{2}}(C'_V + A'_V)$	—	0.76	0.01	0.044

to the tree diagram T and color-suppressed diagram C [12, 26]. Their contributions may be important and can not be ignored. In our present work, we let $a_{E_i,A_i}(i = P,V)$ absorb the relevant formfactors $A_0^{PV}(m_{D_i}^2)$ and take them as global parameters to be fitted, i.e. $a_{E_i,A_i}A_0^{PV}(m_{D_i}^2) \to a_{E_i,A_i}$. In this way, the formula (7)-(10) have the following expressions

$$E_V = \frac{G_F}{\sqrt{2}} V_{q_1q_3} V_{cq_2}^* a_{E_V} 2 f_{D_i} m_{D_i}, \tag{24}$$

$$E_P = \frac{G_F}{\sqrt{2}} V_{q_1q_3} V_{cq_2}^* a_{E_P} 2 f_{D_i} m_{D_i}, \tag{25}$$

$$A_V = \frac{G_F}{\sqrt{2}} V_{q_2q_3} V_{cq_1}^* a_{A_V} 2 f_{D_i} m_{D_i}, \tag{26}$$

$$A_P = \frac{G_F}{\sqrt{2}} V_{q_2q_3} V_{cq_1}^* a_{A_P} 2 f_{D_i} m_{D_i}. \tag{27}$$

It is noted that the formfactors are more appropriate to be viewed as the relative scaling factors that characterize one source of SU(3) flavor symmetry breaking effects in hadronic matrix elements since we take the a_is as free parameters that need to be extracted from experimental inputs in the present method. The relative ratio between the formfactors is what we really care about.

The input values for the light pseudoscalar and vector decay constants are presented in Table 3 [11,27]. These values generally coincide with experiment. According to [11], the decay constants f_η^u, f_η^s, $f_{\eta'}^u$ and $f_{\eta'}^s$ involving in factorization formula should be defined as follow:

$$\langle 0|\overline{u}\gamma^\mu\gamma_5 u|\eta^{(\prime)}(p)\rangle = i f_{\eta^{(\prime)}}^u p^\mu, \tag{28}$$

$$\langle 0|\overline{s}\gamma^\mu\gamma_5 s|\eta^{(\prime)}(p)\rangle = i f_{\eta^{(\prime)}}^s p^\mu. \tag{29}$$

Then the quantities f_η^u, f_η^s, $f_{\eta'}^u$ and $f_{\eta'}^s$ take the formalism

$$f_\eta^u = \frac{f_8}{\sqrt{6}}\cos\phi + \frac{f_0}{\sqrt{3}}\sin\phi, \tag{30}$$

$$f_\eta^s = \frac{2f_8}{\sqrt{6}}\cos\phi - \frac{f_0}{\sqrt{3}}\sin\phi, \tag{31}$$

$$f_{\eta'}^u = -\frac{f_8}{\sqrt{6}}\sin\phi + \frac{f_0}{\sqrt{3}}\cos\phi, \tag{32}$$

Table 5. Preferred solutions of the SU(3) invariant amplitudes in [4] and in the present method. The values in the first entry ($\times 10^{-6}$) are for the magnitudes of the amplitudes and those in the second entry are for the strong phases

		T_V	T_P	C_V	C_P	E_V	E_P	A_V	A_P
The	Case (α)	3.61	7.27	4.06	4.75	0.58	3.05	4.52	4.95
		—	4.3°	172.3°	161.4°	-148.3°	87.4°	-65.4°	-62.2°
Present	Case (α')	3.61	7.29	4.04	4.58	0.71	3.05	4.16	4.67
		—	-1.0°	168.3°	160.0°	-140.8°	87.4°	-69.7°	-68.8°
Values	Case (β')	3.61	5.96	2.71	2.44	3.15	3.05	1.47	1.44
		—	44.2°	-121.6°	-155.8°	-45.4°	87.4°	243.8°	223.7°
[4]		3.61 ± 0.45	6.03 ± 1.15	2.74 ± 0.46	2.44 ± 0.52	3.05 ± 0.18	3.05 ± 0.18	—	—
		—	$(-3 \pm 25)^\circ$	$(-168 \pm 24)^\circ$	$(-156 \pm 12)^\circ$	$(-90 \pm 22)^\circ$	$(88 \pm 11)^\circ$	—	—

Table 6. a_is extracted from the corresponding SU(3) invariant amplitudes in Table 5. The values in the brackets are corresponding to the decay modes $D_s^+ \to PV$. Only the central values are quoted. The case (I) and (II) are the solutions solved out in Sect. 4. The first entry in case (I) and case (II) is for $|a_i|$ and the second entry for the strong phase. The coefficients a_{T_V} and a_{T_P} are corresponding to a_1 extracted from the T_V and T_P diagrams respectively. The coefficients a_{C_V} and a_{C_P} are corresponding to a_2 extracted from the C_V and C_P diagrams respectively

		$D_i \to \pi\rho$	$D_i \to \pi K^*$	$D_i \to K\rho$	$D_i \to KK^*$	$D_i \to \pi\phi$	Case (I)	Case (II)
$\lvert a_{T_V}\rvert$	Case (α)	1.34	1.22 (1.34)	1.05	0.98 (1.07)	(1.21)	1.21	1.21
	[4]	1.34	1.22 (1.34)	1.05	0.98 (1.07)	(1.21)	—	—
$\lvert a_{T_P}\rvert$	Case (α)	1.43	1.32	1.32 (1.46)	1.23 (1.35)	1.12	1.23	1.09
	[4]	1.19	1.09	1.09 (1.21)	1.01 (1.12)	0.93	151.4°	-36.1°
$\lvert a_{C_V}\rvert$	Case (α)	1.50	1.39 (1.50)	1.20	1.09 (1.19)	(1.37)	1.00	0.69
	[4]	1.02	0.93 (1.01)	0.81	0.74 (0.81)	(0.92)	-26.3°	134.1°
$\lvert a_{C_P}\rvert$	Case (α)	0.94	0.86	0.86 (0.96)	0.80 (0.89)	0.72	0.78	0.78
	[4]	0.48	0.45	0.44 (0.49)	0.41 (0.45)	0.38	158.1°	158.1°
$\lvert a_{E_V}\rvert$	Case (α)	0.09 (0.07)	0.10 (0.07)	0.09 (0.07)	0.10 (0.07)	0.09 (0.07)	0.38	0.56
	[4]	0.51 (0.42)	0.51 (0.41)	0.51 (0.42)	0.51 (0.41)	0.51 (0.41)	50.9°	57.3°
$\lvert a_{E_P}\rvert$	Case (α)	0.51 (0.42)	0.51 (0.41)	0.51 (0.42)	0.51 (0.41)	0.51 (0.41)	0.51	0.51
	[4]	0.51 (0.42)	0.51 (0.41)	0.51 (0.42)	0.51 (0.41)	0.51 (0.41)	87.0°	87.0°
$\lvert a_{A_V}\rvert$	Case (α)	0.81 (0.61)	0.75 (0.61)	0.81 (0.61)	0.75 (0.61)	0.75 (0.61)	0.91	0.20
	[4]	—	—	—	—	—	-52.8°	-81.4°
$\lvert a_{A_P}\rvert$	Case (α)	0.83 (0.67)	0.82 (0.67)	0.83 (0.67)	0.82 (0.67)	0.83 (0.67)	0.98	0.27
	[4]	—	—	—	—	—	-53.8°	-75.0°

$$f^s_{\eta'} = \frac{2f_8}{\sqrt{6}}\sin\phi + \frac{f_0}{\sqrt{3}}\cos\phi. \tag{33}$$

By these definitions, the following factorization formalisms are adopted in the $D \to \eta(\eta')V$ transition calculation

$$2C_V(D_i \to \eta V) = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} V^*_{cq_3} a_{C_V} 2(f^u_\eta + f^s_\eta) m_{D_i} A_0^{D_i \to V}(m^2_\eta), \tag{34}$$

$$C_V(D_i \to \eta' V) = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} V^*_{cq_3} a_{C_V} 2(f^s_{\eta'} - f^u_{\eta'}) m_{D_i} A_0^{D_i \to V}(m^2_{\eta'}). \tag{35}$$

The other parameters used in the numerical calculation are the masses of relevant mesons, lifetimes of charmed mesons and relevant CKM matrix elements. We adopt the relevant values given in [9].

For convenience, we may express the complex parameters a_i as

$$a_i = |a_i| e^{i\delta_{a_i}}. \tag{36}$$

The δ_{a_i}s characterize the strong phases. One can always choose $\delta_{a_{T_V}} = 0$ so that all the other strong phases are relative to $\delta_{a_{T_V}}$. There are 15 independent parameters to be extracted from experiments.

From Table 1, one can observe that the decay processes $D_s^+ \to \pi^+\phi$, $D^0 \to \pi^+ K^{*-}$ and $D^0 \to \overline{K}^0\phi$ can be solved to provide information on a_{T_V} and a_{E_P}. These solutions are given as follow

$$|a_{T_V}| = 1.21, \tag{37}$$

$$|a_{E_P}| = 0.51, \qquad \delta_{a_{E_P}} = 87.0^\circ. \tag{38}$$

The next leading order Wilson coefficients $c_1(m_c) = 1.174$ and $c_2(m_c) = -0.356$ in the naive dimensional regularization (NDR) scheme or $c_1(m_c) = 1.216$ and $c_2(m_c) = -0.424$ in the 'tHooft-Veltman (HV) scheme are given in [28] when $\Lambda_{\overline{MS}} = 0.215$GeV. The present value of $|a_{T_V}| = 1.21$ amounts to large N_c in formula (18).

There are three processes $D^+ \to \pi^+\overline{K}^{*0}$, $D^0 \to \pi^0\overline{K}^{*0}$ and $D^+ \to \pi^+\phi$ which can provide us information on a_{C_P}. By combining $D^+ \to \pi^+\overline{K}^{*0}$ and $D^0 \to \pi^0\overline{K}^{*0}$, one can find two sets of solutions for a_{C_P}.

$$(a). \quad |a_{C_P}| = 0.83, \qquad \delta_{a_{C_P}} = 159.4^\circ; \tag{39}$$

Table 7. An example of nonvanished contribution of disconnected diagram SE_P. The second column and the third column are for the central values of a_is in case (I) solution with $a_{SE_P} = 0.03e^{i87.0^\circ}$ as input and in case (II) solution with $a_{SE_P} = 0.03e^{-i93.0^\circ}$ as input respectively.

	Magnitude and Relative Strong Phase in Case (I)	Magnitude and Relative Strong Phase in Case (II)
a_{T_V}	1.21 —	1.21 —
a_{T_P}	1.20 136.0°	1.05 −24.5°
a_{C_V}	0.96 −39.1°	0.64 143.9°
a_{C_P}	0.76 156.7°	0.79 159.5°
a_{E_V}	0.42 38.0°	0.58 67.8°
a_{E_P}	0.43 78.3°	0.59 94.3°
a_{A_V}	0.93 −51.3°	0.24 −80.0°
a_{A_P}	1.00 −52.6°	0.32 −78.2°

$$(b). \quad |a_{C_P}| = 0.43, \qquad \delta_{a_{C_P}} = -154.6^\circ. \tag{40}$$

The solution in case (a) was regarded as the disfavored solution in [3,4] since it induces $|C_P| > |T_V|$ in SU(3) flavor symmetry limit. In the viewpoint of factorization formalism, the case (b) solution means little corrections from the unfactorizable contributions in the method of large N_c in formula (21), while the case (a) solution indicates possible big corrections from the unfactorizable contributions in formula (21). There is no reason to exclude the case (a) solution. Conversely, if one uses the process $D^+ \to \pi^+\phi$, one can obtain $|a_{C_P}| = 0.76$, which indicates that the case (a) solution sounds more reasonable. After performing a fit procedure on these three processes $D^+ \to \pi^+\overline{K}^{*0}$, $D^0 \to \pi^0\overline{K}^{*0}$ and $D^+ \to \pi^+\phi$, we have the following value for a_{C_P}

$$|a_{C_P}| = 0.78, \qquad \delta_{a_{C_P}} = 158.1^\circ, \tag{41}$$

when

$$Br(D^+ \to \pi^+\overline{K}^{*0}) = 1.79\%, \tag{42}$$

$$Br(D^0 \to \pi^0\overline{K}^{*0}) = 2.47\%, \tag{43}$$

$$Br(D^+ \to \pi^+\phi) = 0.64\%. \tag{44}$$

One will find that this solution can provide a good explanation for the process $D^+ \to \overline{K}^0 K^{*+}$ which was considered to be a puzzle in [6].

Combining $D^+ \to K^+\overline{K}^{*0}$ and $D_s^+ \to K^+\overline{K}^{*0}$, we obtain two sets of solutions for a_{A_V}.

$$(\ I). \quad |a_{A_V}| = 0.91, \qquad \delta_{a_{A_V}} = -52.8^\circ, \tag{45}$$

$$(II). \quad |a_{A_V}| = 0.20, \qquad \delta_{a_{A_V}} = -81.4^\circ. \tag{46}$$

Correspondingly, there are also two sets of values for the other parameters which still need to be solved and we will also label them as the case (I) and case (II) correspondingly.

In the process $D_s^+ \to \pi^+\rho^0$, the amplitude $|A_V|$ equals to 6.73×10^{-6} in case (I) solution and equals to 1.48×10^{-6} in case (II) solution. To maintain the experimental upper bound of the process, the phase angle between A_V and A_P should be less than 90° which means the breakdown of the relation $A_V = -A_P$ obtained on the basis of G-parity argument on $q\bar{q}$ resonance [29] in both case solutions. Since ω meson may contain an unknown fraction of $s\bar{s}$ which would permit a T_V amplitude in the process $D_s^+ \to \pi^+\omega$, and most importantly, since ω may induce important contribution from SA_P diagram, we shall take the process $D_s^+ \to \pi^+\rho^0$ instead of $D_s^+ \to \pi^+\omega$ as an input to solve out the a_{A_P}. The Fermilab E791 Collaboration recently reported the measurement $\Gamma(D_s^+ \to \pi^+\rho^0)/\Gamma(D_s^+ \to \pi^+\pi^+\pi^-) = (5.8 \pm 2.3 \pm 3.7)\%$[30]. Though it does not have enough statistic significance, it is still appropriate for us to take the central value in our calculation. The branching ratio $\mathcal{B}(D_s^+ \to \pi^+\pi^+\pi^-) = (1.01 \pm 0.28)\%$ was reported in [9]. If we ignore the difference between the scales characterizing the $c \to s$ and $c \to d$ transitions, we can solve the processes $D_s^+ \to \pi^+\rho^0$, $D^+ \to \pi^+\rho^0$ and get the values

$$(\ I). \quad |a_{A_P}| = 0.98, \qquad \delta_{a_{A_P}} = -53.8^\circ, \tag{47}$$

$$(II). \quad |a_{A_P}| = 0.27, \qquad \delta_{a_{A_P}} = -75.0^\circ. \tag{48}$$

From formula (26) and (27), it is observed that we have the relation $A_P \approx A_V$ in contrast to $A_P + A_V = 0$, which implies that rescattering effects may cause important contributions to break down the relation $A_P + A_V = 0$.

Table 8. Predicted branching ratios for charmed mesons decaying to one pseudoscalar and one vector meson with nonvanished contribution of disconnected diagram SE_P. We take $a_{SE_P} = 0.03e^{i87.0^\circ}$ as input in case (I) solution and $a_{SE_P} = 0.03e^{-i93.0^\circ}$ in case (II) solution.

Meson	Decay Mode	Representation	Experimental $\mathcal{B}(\times10^{-2})$	Present $\mathcal{B}(\times10^{-2})$ case (I)	case (II)	$\mathcal{B}(\times10^{-2})$ in [31]
D^0	K^+K^{*-}	$T'_V + E'_P$	0.20 ± 0.11	0.26	0.26	0.30
	K^-K^{*+}	$T'_P + E'_V$	0.38 ± 0.08	0.43	0.41	0.43
	$K^0\overline{K}^{*0}$	$E'_V - E'_P$	< 0.17	0.02	0.02	0.062
	$\overline{K}^0K^{*0}$	$E'_P - E'_V$	< 0.09	0.02	0.02	0.064
	$\pi^0\phi$	$\frac{1}{\sqrt{2}}(C'_P + SE'_P)$	< 0.14	0.13	0.12	0.11
	$\overline{K}^{*0}\eta'$	$-\frac{1}{\sqrt{6}}(C_P + E_P + 2E_V + 4SE_V)$	< 0.10	0.006	0.015	0.004
	$\eta\phi$	$\frac{1}{\sqrt{3}}(C'_P - 2SE'_P + SE'_V)$	< 2.8	0.03	0.04	0.090
	$\pi^+\rho^-$	$-(T'_V + E'_P)$	—	0.34	0.35	0.57
	$\pi^-\rho^+$	$-(T'_P + E'_V)$	—	0.64	0.64	0.69
	$\pi^0\rho^0$	$\frac{1}{2}(C'_P + C'_V - E_P - E_V)$	—	0.14	0.23	0.12
	$\pi^0\omega$	$\frac{1}{2}(C'_V - C'_P + E'_P + E'_V + 2SE'_P)$	—	0.26	0.16	0.014
	$\eta\omega$	$-\frac{1}{\sqrt{6}}(C'_P + 2C'_V + SE'_V + 4SE'_P)$	—	0.001	0.06	0.20
	$\eta'\omega$	$\frac{1}{2\sqrt{3}}(C'_P - C'_V + 4SE'_V - 2SE'_P)$	—	0.003	0.0009	0.0001
	$\eta\rho^0$	$\frac{1}{\sqrt{6}}(2C'_V - C'_P - SE'_V)$	—	0.10	0.004	0.020
	$\eta'\rho^0$	$\frac{1}{2\sqrt{3}}(C'_V + C'_P + 4SE'_V)$	—	0.0005	0.003	0.008
D^+	$\overline{K}^0K^{*+}$	$T'_P - A'_P$	3.1 ± 1.4	3.62	0.60	1.71
	$\pi^0\rho^+$	$-\frac{1}{\sqrt{2}}(T'_P + C'_V + A'_P - A'_V)$	—	0.21	0.33	0.44
D_s^+	π^+K^{*0}	$-(T'_V - A'_V)$	0.65 ± 0.28	0.53	0.21	0.29
	$K^+\rho^0$	$-\frac{1}{\sqrt{2}}(C'_P + A'_P)$	< 0.29	0.16	0.09	0.29
	$K^0\rho^+$	$-(T'_P - A'_P)$	—	2.87	0.30	1.39
	π^0K^{*+}	$-\frac{1}{\sqrt{2}}(C'_V + A'_V)$	—	0.80	0.016	0.044

When we take $SA_P = 0$ and consider the ideal mixing in ω, we have the branching ratio 40.59% in case (I) solution and 2.50% in case (II) solution for the process $D_s^+ \to \pi^+\omega$, which is much larger than the experimental data $(0.28 \pm 0.11)\%$. Considering that the process may have T_V contribution if ω contains $s\bar{s}$ fraction, the branching ratio can be minimized to 25.89% and 2.37% in the case (I) and (II) solution respectively. To accommodate the experimental data, one has to introduce significant contribution from SA_P in this process, i.e. $SA_P \sim -A_P$.

The values of parameters a_{C_V} and a_{E_V} rely on the processes $D^0 \to \overline{K}^{*0}\eta$, $D_s^+ \to \overline{K}^0K^{*+}$, $D^0 \to \rho^0\overline{K}^0$ and $D^0 \to \omega\overline{K}^0$. By analyzing these processes, the following values are available[1].

$$(I).\quad |a_{C_V}| = 1.00, \qquad \delta_{a_{C_V}} = -26.3^\circ, \quad (49)$$

$$|a_{E_V}| = 0.38, \qquad \delta_{a_{E_V}} = 50.9^\circ; \quad (50)$$

$$(II).\quad |a_{C_V}| = 0.69, \qquad \delta_{a_{C_V}} = 134.1^\circ, \quad (51)$$

$$|a_{E_V}| = 0.56, \qquad \delta_{a_{E_V}} = 57.3^\circ. \quad (52)$$

At last, a_{T_P} is related to the decay processes $D^+ \to \rho^+\overline{K}^0$ and $D^0 \to \rho^+K^-$.

$$(I).\quad |a_{T_P}| = 1.23, \qquad \delta_{a_{T_P}} = 151.4^\circ; \quad (53)$$

$$(II).\quad |a_{T_P}| = 1.09, \qquad \delta_{a_{T_P}} = -36.1^\circ. \quad (54)$$

For convenience, we list in Table 6 the results obtained in our present analysis. A distinct difference between the case (I) and case (II) solution is the contributions from the annihilation diagrams A_V and A_P. The case (I) solution implies important contributions from A_V and A_P, while the case (II) solution signifies relatively small contributions from A_V and A_P. The resulting values imply that nonfactorizable contributions are of significance in $D \to PV$ decays. It is observed that the phase between amplitudes T_V and T_P is large in comparison with that in [4], which leads to the breakdown of the relation $E_P = -E_V$ assumed as an input relation in [3,4]. If the W-exchange amplitudes are dominated by quark-antiquark intermediate states, then a sign flip of E_P relative to E_V will be a consequence of charge-conjugation invariance. In the present calculation, we show that E_V is close to E_P in both cases, which means that the W-exchange amplitudes are not governed by resonant final state interactions. Our analysis is in favor of the claims [12] that the sign flip

[1] There are another set of values $a_{C_V} = 1.12e^{-i85^\circ}$ and $a_{E_V} = 0.27e^{-i160^\circ}$ in case (I) and another set of values $a_{C_V} = 1.22e^{-i155^\circ}$ and $a_{E_V} = 0.08e^{-i172^\circ}$ in case (II) serving as solutions. We discard these solutions as disfavored ones because their predicted branching ratios of some processes have exceeded the experimental upper bounds. Moreover, it is generally believed that $|a_{C_{P,V}}|$ should be smaller than $|a_{T_{P,V}}|$.

Table 9. SU(3) flavor symmetry relations and breaking of the relations. $\lambda = |V_{cs}V_{us}/V_{cs}V_{ud}| \approx 0.226$

SU(3) Symmetry Relations	LHS of Relations in Case (I)	LHS of Relations in Case (II)
$\frac{\lvert\mathcal{A}(D^0\to\pi^+K^{*-})+\sqrt{2}\mathcal{A}(D^0\to\pi^0\overline{K}^{*0})\rvert}{\lvert\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\rvert}=1$	1.0	1.0
$\frac{\lvert\mathcal{A}(D^0\to\rho^+K^-)+\sqrt{2}\mathcal{A}(D^0\to\rho^0\overline{K}^0)\rvert}{\lvert\mathcal{A}(D^+\to\overline{K}^0\rho^+)\rvert}=1$	1.0	1.0
$\frac{\lvert\mathcal{A}(D^0\to\overline{K}^0\phi)-\mathcal{A}(D_s^+\to\pi^+\phi)\rvert}{\lvert\mathcal{A}(D^0\to\pi^+K^{*-})\rvert}=1$	1.00	1.00
$\frac{\lvert\mathcal{A}(D^0\to\pi^+K^{*-})+\mathcal{A}(D^0\to\overline{K}^0\phi)\rvert}{\lvert\mathcal{A}(D_s^+\to\pi^+\phi)\rvert}=1$	0.99	0.99
$\frac{\lvert\mathcal{A}(D^0\to\pi^+K^{*-})-\mathcal{A}(D_s^+\to\pi^+\phi)\rvert}{\lvert\mathcal{A}(D^0\to\overline{K}^0\phi)\rvert}=1$	1.00	1.00
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)\rvert}{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\rvert}=1$	0.87	0.87
$\frac{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)\rvert}{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert}=1$	0.62	0.66
$\frac{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert}{\lvert\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)\rvert}=1$	1.11	1.10
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)-\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)\rvert}{\lvert\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)\rvert}=1$	1.07	0.98
$\frac{\lvert\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)+\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)\rvert}{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert}=1$	0.69	0.90
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert-\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)}{\lvert\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)\rvert}=1$	0.92	1.02
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})+\mathcal{A}(D^+\to K^+\overline{K}^{*0})\rvert}{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\rvert}=1$	0.89	0.89
$\frac{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})-\mathcal{A}(D^+\to K^+\overline{K}^{*0})\rvert}{\lvert\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})\rvert}=1$	0.85	1.28
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})-\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\rvert}{\lvert\mathcal{A}(D^+\to K^+\overline{K}^{*0})\rvert}=1$	1.19	0.81
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})+\mathcal{A}(D^+\to\overline{K}^0K^{*+})\rvert}{\lvert\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)\rvert}=1$	0.82	1.19
$\frac{\lvert\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)-\mathcal{A}(D^+\to\overline{K}^0K^{*+})\rvert}{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})\rvert}=1$	0.93	1.43
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})-\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)\rvert}{\lvert\mathcal{A}(D^+\to\overline{K}^0K^{*+})\rvert}=1$	1.06	0.88
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})+\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)\rvert}{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert}=1$	2.07	0.22
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)\rvert}{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})\rvert}=1$	0.94	0.84
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})\rvert}{\lvert\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)\rvert}=1$	1.07	1.33
$\frac{\lvert\mathcal{A}(D_s^+\to K^0\rho^+)\rvert}{\lvert\mathcal{A}(D^+\to\overline{K}^0K^{*+})\rvert}=1$	1.01	0.77
$\frac{\lvert\mathcal{A}(D_s^+\to\pi^+K^{*0})\rvert}{\lvert\mathcal{A}(D^+\to K^+\overline{K}^{*0})\rvert}=1$	1.19	0.76
$\frac{\lvert\mathcal{A}(D^0\to K^+K^{*-})\rvert}{\lvert\mathcal{A}(D^0\to\pi^+\rho^-)\rvert}=1$	1.21	1.21
$\frac{\lvert\lambda\mathcal{A}(D^0\to\pi^+K^{*-})\rvert}{\lvert\mathcal{A}(D^0\to\pi^+\rho^-)\rvert}=1$	1.05	1.05
$\frac{\lvert\mathcal{A}(D^0\to K^+K^{*-})\rvert}{\lvert\lambda\mathcal{A}(D^0\to\pi^+K^{*-})\rvert}=1$	1.15	1.15
$\frac{\lvert\mathcal{A}(D^0\to K^-K^{*+})\rvert}{\lvert\mathcal{A}(D^0\to\pi^-\rho^+)\rvert}=1$	1.16	1.12
$\frac{\lvert\lambda\mathcal{A}(D^0\to K^-\rho^+)\rvert}{\lvert\mathcal{A}(D^0\to\pi^-\rho^+)\rvert}=1$	1.08	1.06
$\frac{\lvert\mathcal{A}(D^0\to K^-K^{*+})\rvert}{\lvert\lambda\mathcal{A}(D^0\to K^-\rho^+)\rvert}=1$	1.07	1.06

of E_V from E_P is unexplained and the possibility of E_V being close to E_P is not ruled out.

To further test our results, we present the resulting predictions for a variety of charmed meson decay processes in Table 4. It is noted that in our present analysis the SU(3) symmetry breaking effects are not considered in the strong phases, which may bring some deviation from the experimental data. In this sense, our present predictions are in agreement with the existed experimental data. In addition, the predictions for a number of Cabibbo-suppressed modes can be used to test our present analysis in the near future. For comparison, we list the results obtained in [31].

It is of interest that we arrive at a satisfactory explanation for the high branching ratio concerning to the singly Cabibbo-suppressed process $D^+ \to \overline{K}^0 K^{*+}$ if the first case solution is considered. This process was selected out in [6] to be as one of the puzzled processes. It was claimed [6] that there is still not any model to explain the anomalously high branching ratio of this decay mode

Table 10. Breaking of the relations due to mass factors. $\lambda = |V_{cs}V_{us}/V_{cs}V_{ud}| \approx 0.226$

SU(3) Symmetry Relations	LHS of Relations in Case (I)	LHS of Relations in Case (II)
$\frac{\lvert\mathcal{A}(D^0\to\pi^+K^{*-})+\sqrt{2}\mathcal{A}(D^0\to\pi^0\overline{K}^{*0})\rvert}{\lvert\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\rvert}=1$	1.0	1.0
$\frac{\lvert\mathcal{A}(D^0\to\rho^+K^-)+\sqrt{2}\mathcal{A}(D^0\to\rho^0\overline{K}^0)\rvert}{\lvert\mathcal{A}(D^+\to\overline{K}^0\rho^+)\rvert}=1$	1.0	1.0
$\frac{\lvert\mathcal{A}(D^0\to\overline{K}^0\phi)-\mathcal{A}(D_s^+\to\pi^+\phi)\rvert}{\lvert\mathcal{A}(D^0\to\pi^+K^{*-})\rvert}=1$	1.03	1.03
$\frac{\lvert\mathcal{A}(D^0\to\pi^+K^{*-})+\mathcal{A}(D^0\to\overline{K}^0\phi)\rvert}{\lvert\mathcal{A}(D_s^+\to\pi^+\phi)\rvert}=1$	0.95	0.95
$\frac{\lvert\mathcal{A}(D^0\to\pi^+K^{*-})-\mathcal{A}(D_s^+\to\pi^+\phi)\rvert}{\lvert\mathcal{A}(D^0\to\overline{K}^0\phi)\rvert}=1$	1.00	1.00
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)\rvert}{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\rvert}=1$	0.89	0.89
$\frac{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)\rvert}{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert}=1$	0.77	0.61
$\frac{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert}{\lvert\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)\rvert}=1$	1.09	1.10
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)-\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)\rvert}{\lvert\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)\rvert}=1$	1.09	1.05
$\frac{\lvert\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)+\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)\rvert}{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert}=1$	0.71	1.39
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert-\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)}{\lvert\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)\rvert}=1$	0.91	0.95
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})+\mathcal{A}(D^+\to K^+\overline{K}^{*0})\rvert}{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\rvert}=1$	0.88	1.01
$\frac{\lvert\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})-\mathcal{A}(D^+\to K^+\overline{K}^{*0})\rvert}{\lvert\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})\rvert}=1$	0.97	1.00
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})-\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\rvert}{\lvert\mathcal{A}(D^+\to K^+\overline{K}^{*0})\rvert}=1$	1.03	1.00
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})+\mathcal{A}(D^+\to\overline{K}^0K^{*+})\rvert}{\lvert\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)\rvert}=1$	0.97	1.07
$\frac{\lvert\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)-\mathcal{A}(D^+\to\overline{K}^0K^{*+})\rvert}{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})\rvert}=1$	0.99	1.18
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})-\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)\rvert}{\lvert\mathcal{A}(D^+\to\overline{K}^0K^{*+})\rvert}=1$	1.01	0.95
$\frac{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})+\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)\rvert}{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\rvert}=1$	1.30	0.73
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)\rvert}{\lvert\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})\rvert}=1$	0.98	0.93
$\frac{\lvert\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})\rvert}{\lvert\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)\rvert}=1$	1.02	1.09
$\frac{\lvert\mathcal{A}(D_s^+\to K^0\rho^+)\rvert}{\lvert\mathcal{A}(D^+\to\overline{K}^0K^{*+})\rvert}=1$	1.03	1.01
$\frac{\lvert\mathcal{A}(D_s^+\to\pi^+K^{*0})\rvert}{\lvert\mathcal{A}(D^+\to K^+\overline{K}^{*0})\rvert}=1$	1.05	1.00
$\frac{\lvert\mathcal{A}(D^0\to K^+K^{*-})\rvert}{\lvert\mathcal{A}(D^0\to\pi^+\rho^-)\rvert}=1$	1.03	1.03
$\frac{\lvert\lambda\mathcal{A}(D^0\to\pi^+K^{*-})\rvert}{\lvert\mathcal{A}(D^0\to\pi^+\rho^-)\rvert}=1$	1.00	1.00
$\frac{\lvert\mathcal{A}(D^0\to K^+K^{*-})\rvert}{\lvert\lambda\mathcal{A}(D^0\to\pi^+K^{*-})\rvert}=1$	1.03	1.03
$\frac{\lvert\mathcal{A}(D^0\to K^-K^{*+})\rvert}{\lvert\mathcal{A}(D^0\to\pi^-\rho^+)\rvert}=1$	1.04	1.03
$\frac{\lvert\lambda\mathcal{A}(D^0\to K^-\rho^+)\rvert}{\lvert\mathcal{A}(D^0\to\pi^-\rho^+)\rvert}=1$	0.99	0.99
$\frac{\lvert\mathcal{A}(D^0\to K^-K^{*+})\rvert}{\lvert\lambda\mathcal{A}(D^0\to K^-\rho^+)\rvert}=1$	1.05	1.04

and if the high branching ratio is confirmed by more precise experiments it may require new physics to explain it. Based on generalized factorization and including resonance-mediated final state interactions, the branching ratio of the process was calculated to be 1.52% and considered to be unnecessary to introduce new physics [32]. It is seen from our present analysis that the second case solution from our present analysis provides for this process a rather low value comparing to experimental data. If the data for this process is confirmed to have a high branching ratio by more precise experiments, then the first case solution should be a favored one.

The disconnected diagrams are often considered to have no contributions. The present experimental data do not yet contradict with the vanished contributions of these disconnected diagrams except SA_V in $D_s^+ \to \rho^+\eta$ and $D_s^+ \to \rho^+\eta'$ and SA_P in $D_s^+ \to \pi^+\omega$. But as independent parameters, whether these diagrams really vanish or how much they contribute should resort to more experimental data. Here we illustrate as an example to see what im-

Table 11. Breaking of the relations due to formfactors and decay constants. $\lambda = |V_{cs}V_{us}/V_{cs}V_{ud}| \approx 0.226$

SU(3) Symmetry Relations	LHS of Relations in Case (I)	LHS of Relations in Case (II)
$\frac{\|\mathcal{A}(D^0\to\pi^+K^{*-})+\sqrt{2}\mathcal{A}(D^0\to\pi^0\overline{K}^{*0})\|}{\|\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\|}=1$	1.0	1.0
$\frac{\|\mathcal{A}(D^0\to\rho^+K^-)+\sqrt{2}\mathcal{A}(D^0\to\rho^0\overline{K}^0)\|}{\|\mathcal{A}(D^+\to\overline{K}^0\rho^+)\|}=1$	1.0	1.0
$\frac{\|\mathcal{A}(D^0\to\overline{K}^0\phi)-\mathcal{A}(D_s^+\to\pi^+\phi)\|}{\|\mathcal{A}(D^0\to\pi^+K^{*-})\|}=1$	0.97	0.97
$\frac{\|\mathcal{A}(D^0\to\pi^+K^{*-})+\mathcal{A}(D^0\to\overline{K}^0\phi)\|}{\|\mathcal{A}(D_s^+\to\pi^+\phi)\|}=1$	1.05	1.05
$\frac{\|\mathcal{A}(D^0\to\pi^+K^{*-})-\mathcal{A}(D_s^+\to\pi^+\phi)\|}{\|\mathcal{A}(D^0\to\overline{K}^0\phi)\|}=1$	1.00	1.00
$\frac{\|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)\|}{\|\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\|}=1$	0.96	0.97
$\frac{\|\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)\|}{\|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\|}=1$	1.04	1.16
$\frac{\|\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\|}{\|\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)\|}=1$	1.02	1.01
$\frac{\|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)-\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)\|}{\|\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)\|}=1$	0.97	0.92
$\frac{\|\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)+\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)\|}{\|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\|}=1$	1.10	0.55
$\frac{\|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\|-\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)}{\|\sqrt{2}\mathcal{A}(D^+\to\pi^0\rho^+)\|}=1$	1.03	1.07
$\frac{\|\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})+\mathcal{A}(D^+\to K^+\overline{K}^{*0})\|}{\|\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\|}=1$	0.97	0.96
$\frac{\|\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})-\mathcal{A}(D^+\to K^+\overline{K}^{*0})\|}{\|\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})\|}=1$	0.84	1.07
$\frac{\|\lambda\mathcal{A}(D_s^+\to K^+\overline{K}^{*0})-\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})\|}{\|\mathcal{A}(D^+\to K^+\overline{K}^{*0})\|}=1$	1.16	0.82
$\frac{\|\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})+\mathcal{A}(D^+\to\overline{K}^0K^{*+})\|}{\|\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)\|}=1$	0.86	1.10
$\frac{\|\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)-\mathcal{A}(D^+\to\overline{K}^0K^{*+})\|}{\|\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})\|}=1$	0.94	1.19
$\frac{\|\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})-\lambda\mathcal{A}(D^+\to\rho^+\overline{K}^0)\|}{\|\mathcal{A}(D^+\to\overline{K}^0K^{*+})\|}=1$	1.05	0.93
$\frac{\|\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})+\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)\|}{\|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)\|}=1$	1.71	0.42
$\frac{\|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)\|}{\|\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})\|}=1$	0.95	0.91
$\frac{\|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)+\lambda\mathcal{A}(D_s^+\to\overline{K}^0K^{*+})\|}{\|\sqrt{2}\mathcal{A}(D_s^+\to K^{*+}\pi^0)\|}=1$	1.05	1.22
$\frac{\|\mathcal{A}(D_s^+\to K^0\rho^+)\|}{\|\mathcal{A}(D^+\to\overline{K}^0K^{*+})\|}=1$	0.98	0.77
$\frac{\|\mathcal{A}(D_s^+\to\pi^+K^{*0})\|}{\|\mathcal{A}(D^+\to K^+\overline{K}^{*0})\|}=1$	1.14	0.76
$\frac{\|\mathcal{A}(D^0\to K^+K^{*-})\|}{\|\mathcal{A}(D^0\to\pi^+\rho^-)\|}=1$	1.16	1.16
$\frac{\|\lambda\mathcal{A}(D^0\to\pi^+K^{*-})\|}{\|\mathcal{A}(D^0\to\pi^+\rho^-)\|}=1$	1.05	1.05
$\frac{\|\mathcal{A}(D^0\to K^+K^{*-})\|}{\|\lambda\mathcal{A}(D^0\to\pi^+K^{*-})\|}=1$	1.11	1.11
$\frac{\|\mathcal{A}(D^0\to K^-K^{*+})\|}{\|\mathcal{A}(D^0\to\pi^-\rho^+)\|}=1$	1.12	1.09
$\frac{\|\lambda\mathcal{A}(D^0\to K^-\rho^+)\|}{\|\mathcal{A}(D^0\to\pi^-\rho^+)\|}=1$	1.10	1.08
$\frac{\|\mathcal{A}(D^0\to K^-K^{*+})\|}{\|\lambda\mathcal{A}(D^0\to K^-\rho^+)\|}=1$	1.02	1.01

paction the nonvanished SE_P has. The numerical results are presented in Table 7 and Table 8. SE_P is slightly off zero and takes on the direction of E_P in case (I) solution and the inverse direction of E_P in case (II) solution.

5 SU(3) invariant amplitudes

In factorization hypothesis, the coefficients a_is are process independent while the topological amplitudes are process dependent. Phenomenologically, one can also extract the SU(3) invariant amplitudes firstly and then use the formula (3)-(6) and (24)-(27) to calculate the a_is[12]. In this way, the a_is become process dependent yet the amplitudes are process independent. To be more explicit, a_is are SU(3) invariant while the masses, the decay constants and formfactors characterize the SU(3) symmetry breaking effects, which will lead to the SU(3) symmetry breaking in the amplitudes in the viewpoint of the former method. In the latter method, all the quantities, i.e. the a_is, the masses, the decay constants and formfactors are considered to reflect SU(3) symmetry breaking effects in such a way that the amplitudes are kept to be SU(3) invariant.

It is interesting to compare these two methods. For that, we solve out the invariant amplitudes (case(α)) according to the same procedure in the last section and present them in Table 5[2]. In Table 6, we provide the a_is calculated from the invariant amplitudes. It is noted that we can not find a set of solutions indicating small contributions from the annihilation diagrams A_V and A_P in the SU(3) symmetry limit. From the position of factorization hypothesis, $|a_{C_{V,P}}|$ should be smaller than $|a_{T_{V,P}}|$, which indicates that the a_{C_V} in the case (α) solution is not reasonable, while the case (I) and (II) solutions sound reliable.

In [4], the topological amplitudes were extracted out from the Cabibbo-favored $D \to PV$ decay modes firstly and then were expanded to the singly Cabibbo-suppressed modes by multiplying a Cabibbo suppression factor of $\lambda \approx 0.226$. Because there are no enough Cabibbo-favored decay modes to solve out the amplitudes, some assumptions on the relation of E_P and E_V have to be made. In addition, because there are no any more experimental data to constrain the solution, a set of solutions having a small $|C_P|$ were thought to be favored by applying some knowledge of the factorization hypothesis. Such a set of solutions are listed in Table 5. The a_is extracted from this set of solutions are given in Table 6. Obviously, these a_is meet the requirement of factorization hypothesis. But when these amplitudes are extended to the singly Cabibbo-suppressed modes, some processes have inconsistent results with experimental data. For example, with this set of solutions, the process $D^+ \to \pi^+\phi$ has a branching ratio 0.15% which is much smaller than the experimental result 0.61%. These processes require a solution with a large C_P. To be more explicit, we list the results (case (α') and case (β')) extracted out without any assumption on the exchange diagrams, by using the Cabibbo-favored decay modes $Br(D^0 \to \pi^0\overline{K}^{*0})$ and $Br(D^+ \to \pi^+\overline{K}^{*0})$ as inputs to extract the C_P just as did in [4]. The case (α') solution is corresponding to the case (α) solution with a large value of $|C_P|$. While the case (β') is corresponding to the solution favored in [4] that having a small value of $|C_P|$. It is of interest that the case (β') solution really manifests the relation $E_P \approx E_V$.

In short, the extracted amplitudes which are thought to be favored in [4] can provide reasonable a_is, but they can not explain some Cabibbo-suppressed modes. Another set of amplitudes (case (α) or case (α')) can accommodate more experimental data, but they give unreasonable a_is. So the method of extracting a_is from the invariant amplitudes is not reliable to apply to the decay processes in $D \to PV$. The invalidity may result from the fact that the SU(3) symmetry breaking effects in the $D \to PV$ decays are significant to evoke the inconsistency between the SU(3) flavor symmetry and the factorization hypothesis. To obtain reasonable solution of a_is that can cover all experimental data, one should use the method disscussed in the preceding section with considering the SU(3) symmetry breaking effects.

[2] Other solutions are available when we solve the equations. We have selected out this one as a favored solution by using the experimental data in Table 4 as constraints.

6 SU(3) flavor symmetry breaking

As pointed out in [5], SU(3) breaking effects in charmed meson decays appear to be important. In section 5 of this paper, the SU(3) symmetry breaking effects are shown to be large enough to invalidate the method of extracting the a_is from the invariant amplitudes in $D \to PV$ decays. In the SU(3) flavor symmetry limit, there are a number of relations among different decay modes. Based on the above extracted values for the parameters, we can discuss how large are the SU(3) breaking effects in $D \to PV$ decays.

We present these relations in Table 9, as well as the left hand side(LHS) values of the relations in the second and third columns corresponding to the case (I) and case (II) solution respectively. The values in the second and third columns deviating from unit represent the breaking amounts of SU(3) flavor symmetry relations.

It is noted that though these relations deviating from unit reflect the SU(3) flavor symmetry breaking effects, the ones composed of three decay modes and those composed of two decay modes have different sources of breaking terms. To be clear, we take the expressions $\frac{|\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)|}{|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)|}$ and $\frac{|\mathcal{A}(D^0\to K^+K^{*-})|}{|\mathcal{A}(D^0\to\pi^+\rho^-)|}$ as examples.

$$\frac{|\lambda\mathcal{A}(D^+\to\pi^+\overline{K}^{*0})+\sqrt{2}\mathcal{A}(D^+\to\pi^+\rho^0)|}{|\lambda\sqrt{2}\mathcal{A}(D_s^+\to\pi^+\rho^0)|} = \frac{|(T_V+C_P)(D^+\to\pi^+\overline{K}^{*0})-(T_V+C_P-A_P+A_V)(D^+\to\pi^+\rho^0)|}{|A_V(D_s^+\to\pi^+\rho^0)-A_P(D_s^+\to\pi^+\rho^0)|}, \quad (55)$$

$$\frac{|\mathcal{A}(D^0\to K^+K^{*-})|}{|\mathcal{A}(D^0\to\pi^+\rho^-)|} = \frac{|T_V(D^0\to K^+K^{*-})+E_P(D^0\to K^+K^{*-})|}{|-T_V(D^0\to\pi^+\rho^-)-E_P(D^0\to\pi^+\rho^-)|}. \quad (56)$$

In the limit of SU(3) flavor symmetry, the following equations

$$T_V(D^+ \to \pi^+\overline{K}^{*0}) = T_V(D^+ \to \pi^+\rho^0), \quad (57)$$
$$C_P(D^+ \to \pi^+\overline{K}^{*0}) = C_P(D^+ \to \pi^+\rho^0), \quad (58)$$
$$A_V(D^+ \to \pi^+\rho^0) = A_V(D_s^+ \to \pi^+\rho^0), \quad (59)$$
$$A_P(D^+ \to \pi^+\rho^0) = A_P(D_s^+ \to \pi^+\rho^0), \quad (60)$$
$$T_V(D^0 \to K^+K^{*-}) = T_V(D^0 \to \pi^+\rho^-), \quad (61)$$
$$E_P(D^0 \to K^+K^{*-}) = E_P(D^0 \to \pi^+\rho^-), \quad (62)$$

make (55) and (56) equal to one. But from formula (3)–(10), one can find that relations in (57)–(62) are in general not valid. Both the different masses of the charmed mesons and the final light mesons, and the different values of formfactors and decay constants can break the relations in (57)–(62), and thus break the SU(3) flavor symmetry relations in (55) and (56). In addition, by comparing with (55) and (56), one can see that the relations concerning only two decay modes represent the relative SU(3) flavor symmetry breaking amounts of the same diagrams which we call the main diagrams for convenience in later use, while the relations consisting of three decay modes contain additional SU(3) flavor symmetry breaking effects from the other diagrams. So in the relations containing three decay modes, if the SU(3) flavor symmetry breaking contributions of the other diagrams have comparable amounts in comparison with the main diagrams, then the relations will be broken down badly. The main diagram $|A_V - A_P|$ in $D_s^+ \to \pi^+\rho^0$ is relatively small, which usually leads to a significant breaking for the relations when taking $D_s^+ \to \pi^+\rho^0$ as denominator. We present explicitly these relations in the first case solution as follows

$$\frac{|\lambda \mathcal{A}(D^+ \to \pi^+ \overline{K}^{*0}) + \sqrt{2}\mathcal{A}(D^+ \to \pi^+\rho^0)|}{|\lambda\sqrt{2}\mathcal{A}(D_s^+ \to \pi^+\rho^0)|} = 0.62, \quad (63)$$

$$\frac{|\lambda \mathcal{A}(D^+ \to \rho^+ \overline{K}^{0}) + \sqrt{2}\mathcal{A}(D^+ \to \pi^0\rho^+)|}{|\lambda\sqrt{2}\mathcal{A}(D_s^+ \to \pi^+\rho^0)|} = 0.69, \quad (64)$$

$$\frac{|\lambda \mathcal{A}(D_s^+ \to \overline{K}^{0} K^{*+}) + \sqrt{2}\mathcal{A}(D_s^+ \to K^{*+}\pi^0)|}{|\lambda\sqrt{2}\mathcal{A}(D_s^+ \to \pi^+\rho^0)|} = 2.07. \quad (65)$$

It is obvious that SU(3) flavor symmetry analysis is not applicable to such processes.

The first five relations in Table 9 purely concern with the Cabibbo-favored decay modes. The breaking effects in these modes are less than 5%. In particular, the first two relations still conserve because all the decay modes in them form an isospin triangle respectively. Generally speaking, the first case solution and the second case solution have different SU(3) flavor symmetry breaking effects. The breaking effects due to masses and due to formfactors and decay constants in the first case solution can be as large as 21%, while in the second case solution the breaking effects reach 43% (except those relations using the process $D_s^+ \to \pi^+\rho^0$ as denominator).

To be more explicit, we shall separately investigate the SU(3) symmetry breaking effects caused by the masses and by the formfactors and decay constants. To see the SU(3) symmetry breaking effects due to mass difference, we take for the formfactors and decay constants to be in the limit

$$F_1^{D_i \to P}(0) = F_1^{D\to\pi}(0), \qquad A_0^{D_i\to V}(0) = A_0^{D\to\rho}(0),$$
$$f_{D_i} = f_D, \qquad f_P = f_\pi, \qquad f_V = f_\rho. \quad (66)$$

Vice versa, when we mention to the breaking due to formfactors and decay constants, we adopt the relevant values of formfactors and decay constants in Table 2 and 3 while take the masses as $m_P = m_\pi$, $m_V = m_\rho$ and $m_{D_i} = m_D$ and $m_{D_i^*} = m_{D^*}$. We present the numerical results in Table 10 and Table 11. It is seen that the breaking due to masses is up to 12% in the first case solution, while it can reach 18% in the second case solution. The breaking due to formfactors and decay constants is about 16% in the first case solution while in the second case solution it can be as large as 24%.

7 Summary and conclusion

We have studied the $D \to PV$ decays in the formalism of the factorization hypotheses. Two sets of solutions for the parameters are obtained. The first case solution can give satisfactory explanation on the experimental data, especially for the puzzled process $D^+ \to \overline{K}^0 K^{*+}$. The nonfactorizable corrections are likely to be important in $D \to PV$ decays. The relations $E_P = -E_V$ and $A_P = -A_V$ are far away from conservation, which means that the assumption on the contributions of these four diagrams coming mainly from the quark-antiquark intermediate state interactions is not good enough to accommodate the experimental data. It also implies that, to give a consistent analysis on the process $D_s^+ \to \pi^+\omega$ with the experimental result, the disconnected diagram SA_P in the process $D_s^+ \to \pi^+\omega$ should play a significant role. Some relations involving in contributions from $A_P - A_V$ are no longer reliable due to large SU(3) symmetry breaking effects of masses and of formfactors and decay constants. In the formalism of the relations obtained in the SU(3) symmetry limit, the case (I) solution indicates that the breaking effects due to masses and due to formfactors and decay constants are up to 12% and 16% respectively, which can lead to the total breaking amount up to 21% in certain process (except those relations using the process $D_s^+ \to \pi^+\rho^0$ as denominator), when the two symmetry breaking effects due to masses and due to formfactors and decay constants become to be coherently added. In case (II) solution, the breaking effects due to masses and due to formfactors and decay constants are up to 18% and 24% respectively, and the total breaking amount can add up to 43% in certain process (except those relations using the process $D_s^+ \to \pi^+\rho^0$ as denominator). The SU(3) symmetry breaking effects can bring about invalidity when one tries to extract the coefficients a_1 and a_2 from the SU(3) invariant amplitudes and hence should not be ignored in the $D \to PV$ decay modes.

Acknowledgements. We would like to thank Y.F. Zhou for valuable discussion. This work was supported in part by the key projects of Chinese Academy of Sciences, the National Science Foundation of China (NSFC), the BEPC National Lab Opening Project and Associate Scheme at Abdus Salam ICTP, Italy.

References

1. L.L. Chau, H.Y. Cheng: Phys. Rev. D **36**, 137 (1987); Phys. Lett. B **222**, 285 (1989); Phys. Rev. Lett. **56**, 1655 (1986); L.L. Chau: Phys. Rep. **95**, 1 (1983)
2. M. Gronau, O.F. Hernández, D. London, J.L. Rosner: Phys. Rev. D **50**, 4529 (1994); ibid. **52**, 6356, 6374 (1995)
3. J.L. Rosner: Phys. Rev. D **60**, 114026 (1999)
4. C.W. Chiang, Z. Luo, J.L. Rosner: Phys. Rev. D **67**, 014001 (2003)
5. M. Savage, M. Wise: Phys. Rev. D **39**, 3346 (1989); ibid. **40**, 3127(E) (1989)
6. F.E. Close, H.J. Lipkin: Phys. Lett. B **551**, 337 (2003)

7. M. Gronau, J.L. Rosner: Phys. Rev. D **53**, 2516 (1996); A.S. Dighe, M. Gronau, J.L. Rosner: Phys. Lett. B **367**, 357 (1996); ibid. **377**, 325(E) (1996)
8. T. Feldmann, P. Kroll: Eur. Phys. J. C **5**, 327 (1998); T. Feldmann, P. Kroll, B. Stech: Phys. Rev. D**58**, 114006 (1998); Phys. Lett. B **449**, 339 (1999); T. Feldmann, P. Kroll: Phys. Scripta T **99**, 13 (2002)
9. K. Hagiwara et al.: Phys. Rev. D **66**, 010001 (2002) (URL: http://pdg.lbl.gov)
10. L.L. Chau, H.Y. Cheng: Phys. Rev. D **39**, 2788 (1989); L.L. Chau, H.Y. Cheng, T. Huang: Zeit. Phys. C **53**, 413 (1992); H.Y. Cheng, B. Tseng: Phys. Rev. D **59**, 014034 (1999)
11. A. Ali, G. Kramer, Cai-Dian Lü: Phys. Rev. D**58**, 094009 (1998)
12. H.Y. Cheng: Eur. Phys. J. C **26**, 551 (2003)
13. J.G. Korner, K. Schilcher, M. Wirbel, Y.L. Wu: Z. Phys. C **48**, 663 (1990)
14. A. Khodjamirian, R. Rückl: Adv. Ser. Direct High Energy Phys. **15**, 345 (1998)
15. W.Y. Wang, Y.L. Wu: Phys. Lett. B **515**, 57 (2001); ibid. **519**, 219 (2001);
M. Zhong, Y.L. Wu, W.Y. Wang: Int. J. Mod. Phys A **18**, 1959 (2003)
16. J.M. Flynn, C.T. Sachrajda: Adv. Ser. Direct. High Energy Phys. **15**, 402 (1998)
17. A. Abada, D. Becirevic, P. Boucaud, J.P. Leroy, V. Lubicz, F. Mescia: Nucl. Phys. B **619**, 565 (2001)
18. D. Scora, N. Isgur: Phys. Rev. D **52**, 2783 (1995)
19. D. Melikhov: Phys. Rev. D **53**, 2460 (1996); ibid. **56**, 7089 (1997)
20. M. Wirbel, B. Stech, M. Bauer: Z. Phys. C **29**, 637 (1985); M. Bauer, B. Stech, M. Wirbel: ibid. C**34**, 103 (1987)
21. D. Melikhov, B. Stech: Phys. Rev. D**62**, 014006 (2000)
22. S. Gusken et al.: Nucl. Phys. (Proc. Suppl.) **47**, 485 (1996)
23. J.M. Flynn, C.T. Sachrajda: Adv. Ser. Direct. High Energy Phys. **15**, 402 (1998)
24. P. Ball, V. Braun, H. Dosch: Phys. Lett. B **273**, 316 (1991); Phys. Rev. D **44**, 3567 (1991); P. Ball: Phys. Rev. D**48**, 3190 (1993)
25. W.Y. Wang, Y.L. Wu, M. Zhong: Phys. Rev. D **67**, 014024 (2003)
26. P. Zenczykowski: Acta Phys. Polon. B **28**, 1605 (1997)
27. M. Neubert, B. Stech: Adv. Ser. Direct. High Energy Phys. **15**, 294 (1998)
28. G. Buchalla, A.J. Buras, M.E. Lautenbacher: Rev. Mod. Phys.**68**, 1125 (1996)
29. H.J. Lipkin: Argonne National Laboratory Preprint No. ANL-HEP-CP-89-90, published in Heavy Quark Physics, edited by P.S. Drell and D.L. Rubin, AIP Conference Proceedings No. 196, p. 72
30. Fermilab E791 Collabortion, E.M. Aitala et al.: Phys. Rev. Lett.**86**, 765 (2001)
31. F. Buccella, M. Lusignoli, G. Miele, A. Pugliese, P. Santorelli: Phys. Rev. D**51**, 3478 (1995); F. Buccella, M. Lusignoli, A. Pugliese: Phys. Lett. B**379**, 249 (1996)
32. M. Lusignoli, A. Pugliese: hep-ph/0210071